CSC 591, HW: Bayesian Parameter Estimation

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Solution 1:

We know that is known and is unknown. We know that the sample are drawn from

We also know that .

1. The posterior distribution of is given by:

2. We know that the posterior distribution for gaussian is . And from above calculation we know that

3.

From above question we know,

Or we can write the same as

4: from the above equation we can say that the weight average of prior mean and the weighted average of sample mean

5. we can also write the weights as:

Therefor we can say that the weights are inversely proportional to their variances.

6.

Summing up the weights from question 4:

7.

From question 5 equation we can notice that as the value of n increases, the value of and the value of and vice versa.

Therefore, we can say that value of each weight ranges between 0 and 1.

8. we know from question 3 that:

Also, we know that the value of lies between [0,1]. Therefore, we can say that will also lie between [0,1]

9. If σ2 is known, then for the new instance . The posterior predictive is given by:

An alternative proof to this is mentioned in reference: [*https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf*](https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf)

The proof says that:

If are independent, we know and .

10.

Given: and

We know:

Where

R Code to plot:

x <- seq(-10, 10, by = .1)

y <- dnorm(x, mean = 6, sd = 1.5)

y2 <- dnorm(x, mean = 4, sd = 0.8)

y3 <- dnorm(x, mean = 5.7, sd = 0.3)

plot(0,0,xlim = c(0,10),ylim = c(0,1))

lines(x,y,col='red')

lines(x,y2,col='blue')

lines(x,y3,col='green')

